AVL Trees (continued)

Thursday, November 19, 2015 11:33 AM

Problem 7:

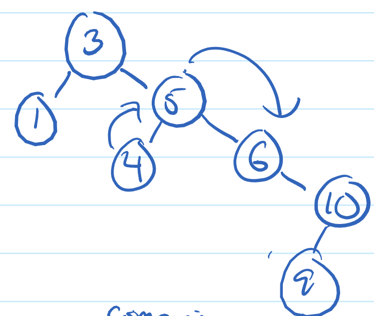
⟨3, 5, 4, 6, 10, 9, 1⟩

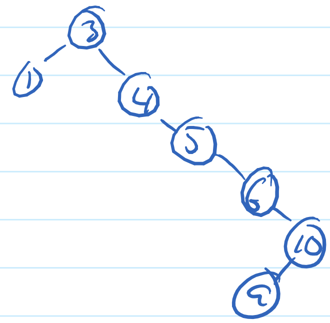
tree.rotate(left, 1)

" " right 5

left 9

After insertions



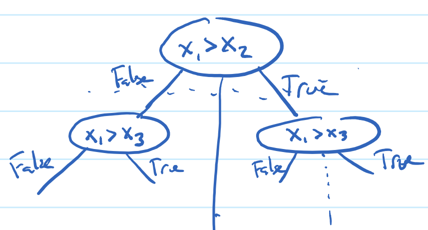


5)

	Comparison Sort
Best	$O(n)$
Avg	$O(n \lg n)$

8) Proof of avg

$x_1\ x_2\ x_3 \ldots x_n$



# of leaves ↓ 2 outputs from 1 decision

of $O(1)$ time operations (# of comparisons) ↙

4 outputs from 2 decisions

8 3

. . . $x_1 \ldots x_2 \ldots$ (leaf) . . . $x_2 \ldots x_1 \ldots$ (leaf) ⟶ $f(x)$

# of leaves $\geq n!$ (all orderings of initial input)

# of leaves $= 2^{f(x)}$

$$2^{f(x)} \geq n!$$

$$\lg 2^{f(x)} \geq \lg n!$$

Stirling's approx

$$O(\lg n! = n \lg n - n \cancel{+ O(\ln n)})$$

$f(x) = \Omega(n \lg n)$

Proof of Best

Insertion sort

1) a) $x^2 \overset{?}{=} \Omega(x)$ True $\Rightarrow$ $x^2 \geq cx$, is there

$x^2 \overset{?}{\geq} x$ $\quad$ $x_0 = 1$ $\quad$ $c = 1$ $\quad$ a c, and an x_0 s.t.

$\forall\, x > x_0$, this is true

b) $2^x = \Theta(x^2)$

False

$2^x = x^2$ $\quad$ $c = 1$ $\quad$ $x_0 = 10$

c) $\lg(x) = \Theta(\log x)$ True

$\log_a x = \dfrac{\log_b x}{\log_b a} \Rightarrow c$ $\qquad$ $\dfrac{\log 8}{\log 2} = 3 \Rightarrow \log_2 8$

$x \quad x^2$

2) a) If $f(x) = \Omega(g(x)) \Rightarrow g(x) = \Theta(f(x))$

$f(x) \geq g(x) \Rightarrow g(x) = f(x)$?

$3 \geq 2 \qquad 2 \overset{?}{=} 3$

$x^2 = \Omega(x)$, but $x \neq \Theta(x^2)$

b) If $f(x) = \Omega(g(x)) \overset{?}{\Rightarrow} g(x) = O(f(x))$ True

$f(x) \geq g(x) \overset{?}{\Rightarrow} g(x) \leq f(x)$

c) If $\boxed{f(x) = O(g(x))} \overset{?}{\Rightarrow} g(x) = O(f(x))$ False

$f(x) \leq g(x) \overset{?}{\Rightarrow} g(x) \leq f(x)$

Let $f(x) = x$ $\qquad$ $x = O(x^2)$, but $x^2 \neq O(x)$

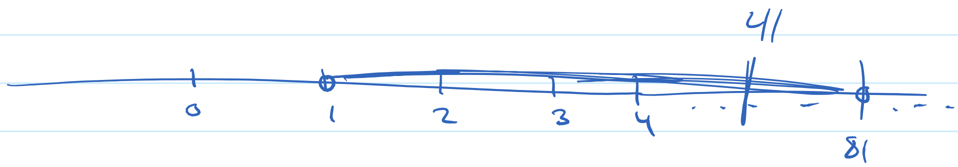
$g(x) = x^2$

Lab Exam (Binary Search)

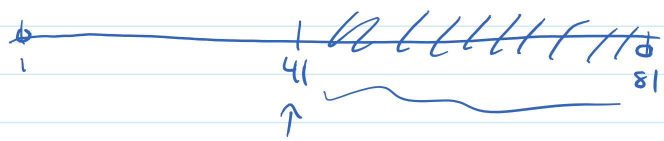
4^{th} roots $\qquad$ 81

4th roots

81



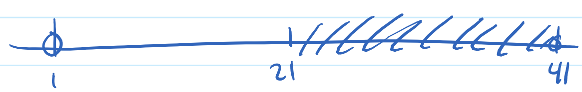
$$\frac{1+81}{2} = 41$$



Guess

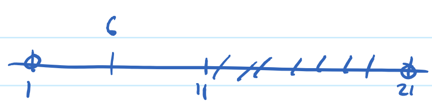
$41^4 \stackrel{?}{=} 81$

$41^4 \approx 2.8e6$



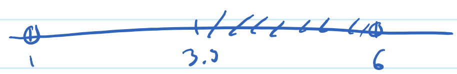
$21^4 \stackrel{?}{=} 81$

$21^4 = 1.94e5$



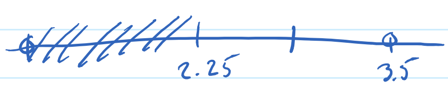
$11^4 \stackrel{?}{=} 81$

$11^4 \approx 14{,}000$



$3.5^4 = 150$

.0001



$2.25^4 = 25.6$

```
private static final double TOLERANCE = 0.00001;
   root (double num) {
       if (num <= 0) {
          ... (return)
```

```
        .... (return)
    }
    if ( num < 1 ) {
        root( num, (num+1)/2, num, 1)
    }
    root( num, (num+1)/2, 1, num)
}
```

$.3 == (.2 + .1)$

```
root ( num, current, low, high) {
    if (Math.abs(Math.pow( current, 4 ) - num) ≤ TOLERANCE) {
                return current;
    }  ← print
    if ( Math.pow(current, 4) > num) {
        // Too high
        return root ( num, (low+current)/2, low, current);
    }
    return root( num, (current+high)/2, current, high);

}
```

0.6000000000

```
printf ("%d \n", 1);
printf ("%s \n", "Hello, world");
```

"%f"

"%.4f"

```
String.format("%.5f", double)
```

type printable

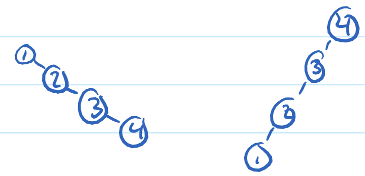
```
System.out.printf("%s  %.5f",  ,  )
```

str double

Math.ceil 2.99999 → 3
Math.floor 2.99999 → 2

Balanced BST

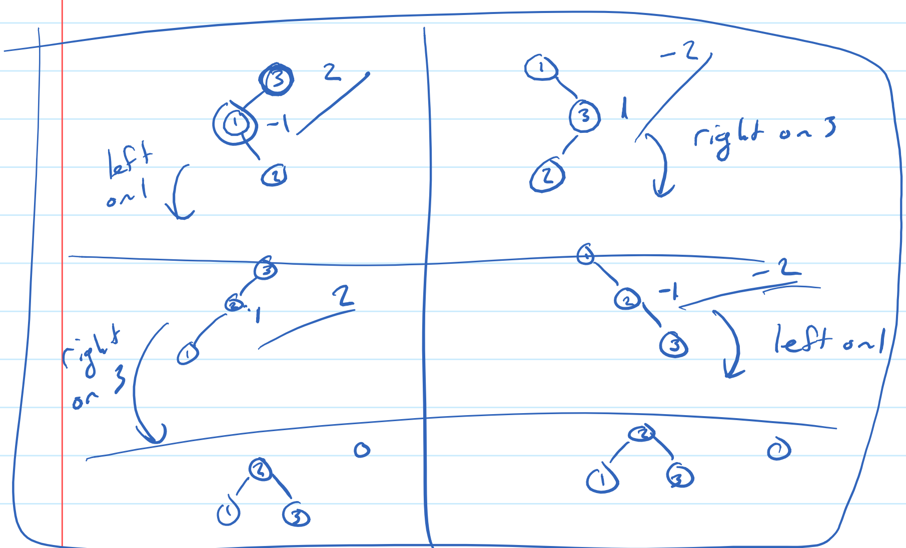
1 2 3 4



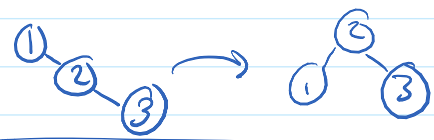
"Balancing" → balance factor = size(left) - size(right)

$\boxed{\text{Want } |\text{balancefactor}| \le 1}$

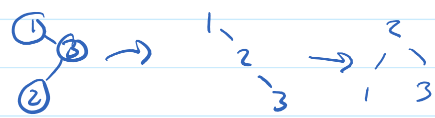
Perfect balancing is difficult



⟨1, 2, 3⟩



⟨1, 3, 2⟩



E extends Comp<Object>

E extends Comp<E>

AVLTree<....>

(E elem)

elem.compareTo(....)